**Recursive Trace for gdc(8, 6)**

gdc(8,6)

gdc(6,2)

gdc(2,0)

return 2

return 2

return 2

**Analyzing hanoi for time complexity:**

* Time to move n disks = T(n)
* There are two recursive calls for n – 1 disks and one constant
  + Constant = time to move one disk = 1
* T(n) = 2 T(n - 1) + 1

Analysis

* T(1) = 1
* T(2) = (2 \*1) + 1
* T(3) = (4 \* 1) + (2 \*1) + 1
* T(n) = (2n-1+ 2n – 2 + … + 21 + 20) \* 1 = (2n – 1) \* 1

**Analyzing gdc for correctness:**

* Base Case: b == 0, gdc(a, b) = gdc(a, 0) = a
* Inductive Case:
  + an and bn = a and b in gdc(a, b)
  + q and r = the quotient and remainder of dividing bn by an
  + gdc(an , bn) = gdc(bn, an) = gdc(bn, qbn + r) = gdc(bn, r)
    - an = qbn + r
    - 0 <= r < bn
    - r = an % bn
  + Recursive call
    - gdc(an -1, bn – 1) = gdc(bn, r)